

Trajectory of a Particle around Black Hole

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point1[m1_] := Module[{a}, a = m1; a/100];
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Manipulate[
Module[{trajectoryplot, blackhole1},
sol[m1_, pt1_] := Quiet[
NDSolve[{x'[t] == ux[t] / (1 + m1/Sqrt[(x[t] - pt1[[1]])^2 + (y[t] - pt1[[2]])^2]),

y'[t] == uy[t] / (1 + m1/Sqrt[(x[t] - pt1[[1]])^2 + (y[t] - pt1[[2]])^2]),

ux'[t] == ((1 + 2 (ux[t]^2 + uy[t]^2)) *
(-((m1 (x[t] - pt1[[1]])) / ((x[t] - pt1[[1]])^2 + (-pt1[[2]] + y[t])^2)^(3/2))) -
ux[t] * (ux[t] (-((m1 (-pt1[[1]] + x[t])) /
((-pt1[[1]] + x[t])^2 + (-pt1[[2]] + y[t])^2)^(3/2))) +
uy[t] (-((m1 (-pt1[[2]] + y[t])) /
((-pt1[[1]] + x[t])^2 + (-pt1[[2]] + y[t])^2)^(3/2)))))) /
((1 + m1/Sqrt[(x[t] - pt1[[1]])^2 + (y[t] - pt1[[2]])^2])^2),

uy'[t] == ((1 + 2 (ux[t]^2 + uy[t]^2)) *
(-((m1 (-pt1[[2]] + y[t])) / ((-pt1[[1]] + x[t])^2 + (-pt1[[2]] + y[t])^2)^(3/2))) -
ux[t] * (ux[t] (-((m1 (-pt1[[1]] + x[t])) /
((-pt1[[1]] + x[t])^2 + (-pt1[[2]] + y[t])^2)^(3/2))) +
uy[t] (-((m1 (-pt1[[2]] + y[t])) /
((-pt1[[1]] + x[t])^2 + (-pt1[[2]] + y[t])^2)^(3/2)))))) /
((1 + m1/Sqrt[(x[t] - pt1[[1]])^2 + (y[t] - pt1[[2]])^2])^2),

x[0] == xi, y[0] == yi, ux[0] == ux0, uy[0] == uy0}, {x[t], y[t], ux[t], uy[t]},
{t, 0, 200}, "ExtrapolationHandler" -> {Indeterminate &}];

trajectoryplot = ParametricPlot[Evaluate[{x[t], y[t]} /. sol[m1, pt1]],
{t, 0, 200}, PlotStyle -> {{Hue[color], Dashed, Thickness[0.01]}}] /.
Line[x_] -> {Arrowheads[{0.04, 0}], Arrow[x]};
```

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blackhole1 = Graphics[{Opacity[.7], Black, PointSize[point1[m1]], Point[pt1]};

Show[backgroundPlot[pt1, pt2, check], trajectoryplot,
  blackhole1, blackhole2, PlotRange → {{-8, 8}, {-8, 8}}],
{{xi, 0, Row[{"initial ", Style["x", Italic], " position"}]},
 -8, 8, .1, ImageSize → Tiny, Appearance → "Labeled"},
{{yi, 0, Row[{"initial ", Style["y", Italic], " position"}]}, -8,
 8, .1, ImageSize → Tiny, Appearance → "Labeled"}, Delimiter,
{{ux0, 0, Row[{"initial ", Style["x", Italic], " momentum"}]},
 -10, 10, .1, ImageSize → Tiny, Appearance → "Labeled"},
{{uy0, 0, Row[{"initial ", Style["y", Italic], " momentum"}]},
 -10, 10, .1, ImageSize → Tiny, Appearance → "Labeled"}, Delimiter,
{{m1, 1, "mass black hole 1"}, 1, 10, 1, ImageSize → Large, Appearance → "Labeled"},
Delimiter,
Style["location of black holes"],
{{pt1, {0, 1}, "black hole 1"}, {-8, -8}, {8, 8}, ImageSize → Small},

Delimiter,
{{check, False, "show density plot?"}, {True, False}},
{{color, .5, "trajectory color"}, .01, 1, ImageSize → Tiny},
SynchronousUpdating → False,
SaveDefinitions → True, TrackedSymbols →
{xi, yi, Delimiter, ux0, uy0, Delimiter, m1, m2, Delimiter, pt1, pt2, check, color}
]

```

initial x position

initial y position

initial x momentum

initial y momentum

mass black hole 1

location of black holes

black hole 1

show density plot? ☐

trajectory color